Uniting the wave-particle dual natures via a nonlocal-hopping particle: Likely possibility of measuring a single unknown quantum state

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Motivated by uniting the apparently contradictory dual behaviors exhibited by an atom/photon viz., wave and particle, here we propose a new theory/interpretation of quantum mechanics (which is consistent with the known experimental results) which predicts the existence of a particle which can hop/jump in a nonlocal/discontinuous and random fashion, and we show that there are already strong experimental (and also Advaitha/non-dual philosophical) evidences in favor of the same. Existence of a nonlocal-hopping particle may have profound implications on both technology and fundamental research e.g., it might be possible to measure a single unknown quantum state by exploiting its ability to emit/scatter photons from different regions of spatial superposition (we propose an experiment to test the same). Further it may point towards the possible existence of an underlying absolutely empty and only indirectly observable quantum nonlocal wave/field from which spacetime itself (and also matter) might be emerging (otherwise nonlocal-hopping is not logical).

Wave-particle duality:

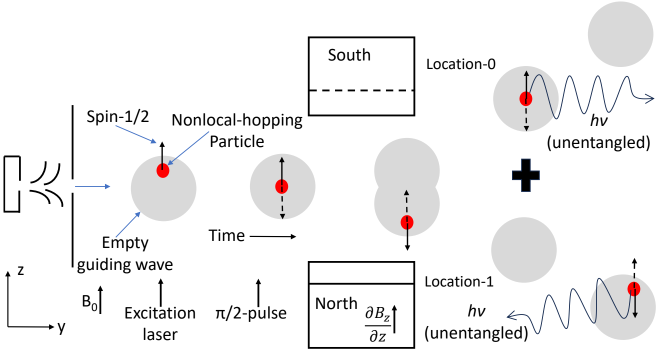

FIG. 1. Schematic diagram depicting the essence of nonlocal-hopping particle interpretation: An internally highly-excited massive single atom/molecule, spontaneously emitting distinguishable photons (which are not entangled with the atom/molecule because of no hopping during a given spontaneous emission event) in a cascaded fashion from different regions of spatial superposition by virtue of nonlocal-hopping wherein the mean hopping time $\bar{\tau} \propto 1/\lambda_{dB}$. We do not detect photons until at least two unentangled photons are emitted (entangled photons are not shown).

Particle natures exhibited by a quanton: 1. The plane monochromatic matter-wave $\exp(ipx/\hbar)$ tells, by virtue of de Broglie relation $p = h/\lambda$, that particle nature is associated with a quanton [56] (i.e., any quantum object like an atom, photon, electron etc.) apart from wave nature (this is also justified by the fact that $\exp(ipx/\hbar)$ satisfies proper energy-momentum relation even in the relativistic scenario [126]). But it do not tell if it is a nonlocal-hopping particle or a non-hopping particle (like in Bohmian mechanics (BM) [20]). Note that no such particle is associated with a classical electromagnetic (EM) plane monochromatic wave $\sim \mathrm{Real}(\exp(ikx))$ at time $t = 0$ where $k = 2\pi/\lambda$ [88]. 2. Consider Young's double slit experiment with a single quanton [11]. We obtain a single well localized particle-like spot on the screen but not interference fringes, however faint/weak [42]. This implies that the quanton is not a pure wave like classical EM wave. Hence the quanton has particle nature as well [112]. 3. Absence of electrostatic self-repulsion of electron in an Hydrogen atom (which is mathematically taken-care/implied by the linearity of Schrödinger equation (SE)) [67, 68] is a consequence of its particle nature. Note that if the electron were a pure wave wherein the charge is statically/uniformly delocalized/distributed across the spatial points allowed by the wave function (like in some interpretations of BM [23]) then it should have repelled by itself. If the particle (associated with the electron) were a classical particle (wherein the charge is always localized) like in BM (this is justified by the concept of empty wave within BM [55, 64, 81, 85, 141, 152]) then it contradicts the experimental fact that there is no radiation [42, 88] due to acceleration [86] of electron implied by the Bohmian guidance equation (see postulate-3 below), and also it fails to account for the experiments which demand delocalization of charge (see point-2, next section). Only a nonlocal-hopping particle (wherein the charge, mass etc., are dynamically delocalized as shown in Fig. 1) is consistent with all the preceding experimental facts (these motivate postulates-1,3 below). 4. Consider gravitationally induced neutron[43]/atom[91, 99, 127] interference. These experiments demand mass of a single neutron/atom to be spatially delocalized over distances $\ggg$ fermi. If quarks/nucleons were pure wave then static delocalization of neutron/atom mass (like in some interpretations of BM [23]) contradicts the very short range ($\sim$ fermi) nature of strong nuclear force [15, 42] as it implies separating the quarks/nucleons [15] by distances $\ggg$ fermi. Moreover the experiments [43, 91, 99, 127] demand the full (but not a fraction of) mass to be present in each of the two interfering paths (this is a consequence of linearity of SE) (postulate-4). If we assume that the same single neutron/atom itself exists simultaneously in both the interfering paths always, then also we encounter contradictions as explained below. This can be satisfactorily explained only via the concept of dynamic delocalization of neutron/atom mass. 5. We cannot explain the photoelectric effect, Compton scattering [15], and scattering of light by light in vacuum [88] by considering light as classical EM wave but it requires the concept of a particle (photon). 6. The experiments discussed in [12, 73] show that one can observe (via commuting observables) both wave and particle natures of a quanton, using the same given experimental arrangement (the weak value based quantum cheshire-cat experiment [109] also demonstrates a similar thing). Even Wheeler's delayed choice experiment shows that wave and particle should exist together [109]. 7. The experiments discussed in [46, 87] cannot be explained satisfactorily without the concept of a particle. 8. Consider a free electron/neutron whose wave packet is localized in space and moving along x-axis. Here spin angular momentum of the electron/neutron also traverses along x-axis even though there is no coupling between spin and spatial degree(s) of freedom (d.o.f) wavefunctions/state-vectors (latter is justified by an NMR-gyroscope [89]). To explain this we need the concept of a particle whose properties (mass, spin, charge etc.) are localized in space. (See supplementary material for details.)

Wave natures exhibited by a quanton: 1. Interference, tunneling, quantum contextuality and nonlocality, refraction, diffraction etc. 2. Without assuming spatial delocalization of particle properties like charge, magnetic-dipole-moment/spin, mass etc. (which is a consequence of wave nature of a quanton), one cannot satisfactorily explain the following experiments: Aharonov-Bohm (AB) (charge) [3, 23, 149] and Aharonov-Casher (AC) (spin) [39, 159] effects, neutron interferometry experiments (mass [43] and spin [108]), Deutsch quantum computing algorithm wherein we selectively apply a half-wave plate in one of the two arms of the Mach-Zehnder interferometer [137], and Schrödinger's (effective) charge density hypothesis which has support from quantum chemistry experiments [140] and protective measurement of a single Hydrogen atom wave function [67] (postulate-1). 3. In an Hydrogen atom, existence of stable (against radiation from an accelerating charge) states of the electron is also a consequence of its wave nature [42] (postulate-3). 4. Nonlocal momentum transfer in vector AB effect [4] (this in turn justifies the possibility of nonlocal mass transfer via nonlocal-hoppings in postulate-2, below). (See supplementary material for details.)

Nonlocal-hopping particle (i.e., particle without hidden variable): To satisfactorily account for the apparently contradictory behaviors of a quanton [42] discussed above, it is necessary to assume that a quanton has both wave and particle natures, and they exist together/simultaneously. This in turn requires the quanton to possess some property different from that of wave (symmetry breaking) for a positive and finite time period (this is sufficient). Otherwise quanton will be indistinguishable from pure wave (symmetric state) always, and consequently there can be no concept of particle at all. The experimental observations discussed above (which correspond to a square-integrable/physical state [42] of a quanton), demand localization of mass, charge etc., in position basis for a positive time period (we refer to it as the definite value of position of an apparent/hypothetical particle). This in turn implies existence of a trajectory and hence particle possessing definite value of momentum simultaneously with position for a positive time period. These observations motivate us to put forth the following four postulates of nonlocal-hopping particle interpretation (NPI):

- 1. There exists a quantum particle without hidden variable (which is just a quanta of excitation of an underlying empty quantum guiding wave) having definite values of both position $\vec{r}$ and momentum $\vec{p}$ simultaneously during any given interval of time $t'_i < t < t'_{i+1}$ where $t'_{i+1} = t'_i + \tau, 0 < \tau < \infty$ is a random variable; particle properties like mass/energy, charge, magnetic dipole moment etc., are localized at $\vec{r}$; and the state of the particle is described by a point $(\vec{r}, \vec{p})$ in the phase-space, $i = 1, 2, \ldots$. At $t = t'_i$, the quanton/particle hops/jumps discontinuously/instantaneously/nonlocally in both position and momentum bases simultaneously (by exchanging energy and/or momentum with a quantum potential), and consequently it has no definite values of both position and momentum, and hence no particle but only an empty wave exists $\forall i$. This in turn precludes the possibility of hidden variable description like in BM.

- 2. At any given time t, the state of an underlying, unmanifest, completely/absolutely empty (i.e., devoid of all the particle properties), only indirectly observable (via hopping particle; this is justified by the experiments [63, 70, 158] which have failed to observe directly the empty wave of de Broglie and Bohm which is objectively real like classical EM wave [17, 87] and also by Young's double slit experiment with single electrons [11]), and abstract quantum nonlocal guiding wave is described by the complex wave function $\psi(\vec{r}, t)$. And the probability of hopping particle being in the closed interval $[\vec{r}, \vec{r} + d\vec{r}]$ at time $t \in (t'_i, t'_{i+1})$ is given by $|\psi(\vec{r}, t)|^2 d^3r \forall i$.

- 3. During any given open interval (t'_i, t'_{i+1}), particle apparently decouples from wave [because there is no Bohmian kind of coupling between wave and particle implied by the guidance equation $\vec{v} = (1/m)\vec{\nabla}S(\vec{r},t)$ where $\psi(\vec{r},t) = \mathcal{R}(\vec{r},t)e^{iS(\vec{r},t)/\hbar}$, and $m, \vec{v}$ are mass, velocity respectively of the particle [20]] such that if the quanton is in a bound (an unbound) square-integrable state then the particle evolves like a classical-free particle (classical particle, in general) $\forall i$. Consequently if a phenomena (e.g., spontaneous photon emission) is restricted to a given interval (t'_i, t'_{i+1}) then the state of the quanton may be described by a point in the phase-space (neglecting the guiding wave), and consequently there will be no entanglement between the quanton and the photon it emits (Eq. (10)). In case of a single particle, if both scalar and vector potentials [42] are zero then wave nature will be manifest only over time scales $\gg \bar{\tau}$ (mean of τ) via nonlocal-hoppings. Otherwise there is possibility of some kind of wave nature being manifest even during a given interval (t'_i, t'_{i+1}) e.g., absence of radiation due to acceleration of electron in an Hydrogen atom, quantum mechanical tunneling, Bell nonlocal correlation, nonlocal momentum transfer in vector AB effect [4] etc.

- 4. During (t'_k, t'_{k+1}), the wave evolves according to NPI SE: $i\hbar\frac{d}{dt}|\psi(t)\rangle = H_N|\psi(t)\rangle$ where $|\psi(t)\rangle = \sum_j \alpha_j|\varphi_j(t)\rangle$, $H_N = b(E(t))H_U$, $H_U|\varphi_j(t)\rangle = E_j|\varphi_j(t)\rangle$, $b(E(t))$ is nonzero only at the location $E(t) \in \{E_j\}$ where the particle is present in energy eigenbasis, and the particle evolves according to Newton's equation (subject to postulate-3) $\forall k$. Discontinuous jumps of the particle in position, momentum, energy etc., bases at $t = t'_k \forall k$ are governed by master equations similar to (13). $\Rightarrow$ $|\psi(t'^{-}_{k+1})\rangle = N|\psi(t'^{+}_{k})\rangle$ where $N = \exp(-iH_N\tau/\hbar)\forall k$.

Similar postulates hold for other d.o.f like spin, as well. Postulate-1 implies existence of a continuous and smooth trajectory of the particle during any given interval $(t'_i, t'_{i+1})\forall i$. (Note that a classical particle i.e., a particle with hidden variable not guided by an empty quantum nonlocal wave corresponds to $-\tau \leq t \leq \tau, \tau \to \infty$ in the above definition of a particle without hidden variable.) We call the nonlocal-hopping particle as a dynamically delocalized particle. Note that in NPI, because of eliminating hidden variable (via nonlocal hopping), there is scope for randomness to emerge from within unlike in BM. Hence we assume that τ is a random variable (note that to eliminate hidden variable we do not require τ be a random variable). This is justified by the fact that no point in space and time are preferred for hoppings. Hence hoppings must be intrinsically random. We call this as NPI. This explains the origin of randomness in measurement outcomes and also reproduces Born's stochastic interpretation of the wave function. Further, within NPI, as randomness arises from within as a consequence of eliminating hidden variable, we may say that randomness is also a consequence of wave nature of a quanton.

Note that Born's statistical interpretation of $\psi(\vec{r},t)$ [75] follows naturally from the above interpretation of $\psi(\vec{r},t)$ within NPI. Conversely, if we assume that Born's stochastic interpretation intrinsically holds even for a single quanton (but not just for an ensemble) then NPI follows from it [129] (this is justified by similarities/connections between Schrödinger's charge density hypothesis and Born's interpretation of wave function [66, 140], and NPI [67]). Note that Born's interpretation of wave function is independent of the wave function collapse postulate which seems to be not required within NPI. (See supplementary material for details.)

Limiting cases of NPI: In the limit $\tau \to 0$ $[\infty]$ with $-\tau \leq t \leq \tau$, NPI $\to$ unexcited/unmanifest quantum world (UQW) wherein only an absolutely empty wave exists [classical particle/mechanics with an underlying unmanifest empty wave which has completely decoupled from the particle]. Note that even in the limit $\tau \to \infty$, NPI $\nrightarrow$ BM because of the absence of Bohmian kind of coupling between wave and particle and because the dynamics is nonlinear. However for $\bar{\tau} \gg 1$, there are no hoppings on practical time scales within NPI and hence we may consider NPI $\approx$ BM (because then Bohmian coupling between wave and particle, and evolution under NPI SE have no significant/observable effects). Consequently, within NPI, there seems to be no need of wave function collapse (because no need of collapse within BM [87, 152]) to account for the macroscopic/classical world we observe. (See supplementary material for details.)

Defining $\bar{\tau}$: The preceding observation shows that $\bar{\tau}$ is a measure of particle nature of a given quanton and hence it should be inversely proportional to its de Broglie wavelength i.e.,

$$\bar{\tau} = \frac{\kappa}{\lambda_{dB}} \tag{1}$$

where κ may depend on the interaction and the properties of the quanton, and we have assumed that the quanton is in an eigenstate of momentum. When the quanton is not in an eigenstate of momentum then we define $\bar{\tau} = \kappa/\bar{\lambda}_{dB} = \kappa\bar{p}/h$ where $\bar{p} = \langle|\vec{P}|\rangle_\psi$ or $\sqrt{\langle\vec{P}^2\rangle_\psi}$; $\vec{P}, \psi$ are momentum-operator, wave function respectively corresponding to center of mass (c.m) d.o.f of the quanton, and h is the Planck's constant. $\Rightarrow$ larger the wave nature of a quanton, larger will be the mean frequency of hoppings ($\bar{\nu} = 1/\bar{\tau}$) which is the main source of manifesting wave nature within NPI. Note that in Eq. (1) we have defined only the mean of τ whereas its probability distribution has to be found experimentally via photons emitted/scattered from different locations by a quanton in spatial superposition (see supplementary material for details). This in turn implies that during any given period of time $\Gamma \gg \bar{\tau}$, the particle spends (on an average) $\int_0^\Gamma |\psi(\vec{r},t)|^2 d^3r dt$ [Time] in the region $[\vec{r}, \vec{r}+d\vec{r}]$. Further note that instantaneous/dynamic mean frequency of hoppings may be defined as $\bar{\nu} = h/(\kappa p)$ where p is the magnitude of momentum of the hopping particle at a given instant of time (excluding the hopping instants $t = t'_i \forall i$).

κ may depend on EM, weak, strong, and gravitational interactions: Theoretically predicted shortest possible life-time of a bound excited energy eigenstate of a charged particle (interacting with quantized EM field vacuum) of rest-mass m_o and charge q is given by $\bar{\mathcal{T}}_{spo,min,th}(m_o, q|EM) \approx 3\hbar^2/(Q^2 m_o c)$ where $Q^2 = q^2/(4\pi\epsilon_0)$, ϵ_0 is the permittivity of free space, c is the speed of light in vacuum [45]. $\Rightarrow$ for an electron $\bar{\mathcal{T}}_{spo,min,th}(e^-|EM) \approx 0.53\ as$ [45] where attosecond $(as) = 10^{-18}s$. The experimentally obtained lifetimes of an excited electron in the context of Auger effect [13, 15] are $\sim fs$ [113] and $\sim 200as$ [148]. The theoretically calculated lifetime of Fm^{99+} ion is $16as$ [151]. These are close to but significantly greater than $0.53as$ and hence validate the theoretical lower bound. To account for this within NPI, we require $\bar{\tau}(e^-|EM) \leq \bar{\mathcal{T}}_{spo,min,th}(e^-|EM) \sim as$ wherein the upper-bound corresponds to a single hopping during the deexcitation process (but note that within SQM, deexcitation is a continuous process [45] and hence it should involve superposition of ground and excited states). This can be satisfied provided we define (based on dimensional analysis) $\kappa(e^-|EM) \lesssim 6\pi\hbar^3/(Qm_oc)^2 \sim 10^{-30}s.m$.

Similarly, for a proton we obtain $\bar{\mathcal{T}}_{spo,min,th}(p^+|EM) \approx 289ys$ where yoctosecond $(ys) = 10^{-24}s$. But the experimentally observed value (i.e., half life of ${}^5_1$H nucleus) is $\bar{\mathcal{T}}_{spo,min,ex}(p^+|EM, weak, strong) \approx 86ys$ [98] which is significantly less than $289ys$ and hence clearly violates the theoretical lower bound. This in turn implies that for a given quanton, the shortest possible lifetime of an excited bound state depends on the dominant interaction (EM, weak, strong, gravitational) involved in the deexcitation/decay process and $\bar{\mathcal{T}}_{spo,min,ex}(m_o,q|EM) > \bar{\mathcal{T}}_{spo,min,ex}(m_o|weak, strong)$. Consequently $\bar{\tau}$ (and hence κ) may also depend on the dominant interaction involved in the phenomena under consideration and $\bar{\tau}(m_o,q,\lambda_{dB}|EM) > \bar{\tau}(m_o,\lambda_{dB}|weak,strong)$. And based on the relative strengths of different interactions [15], we assume that $\bar{\tau}(m_o,\lambda_{dB}|gravitational) \ggg \bar{\tau}(m_o,q,\lambda_{dB}|EM) \gg \bar{\tau}(m_o,\lambda_{dB}|weak,strong)$ (assuming the shortest possible nuclear half life is significantly $< 86ys$) which in turn implies $\kappa(m_o|gravitational) \ggg \kappa(m_o,q|EM) \gg \kappa(m_o|weak,strong)$. Further, to account for the fact that wave nature $(\lambda_{dB},\bar{\nu})$ exhibited by neutral particles is inversely proportional to their rest-mass (at a given speed $0 < v \ll c$; note that this is in general not true in case of charged particles as evident from $\kappa(e^-|EM) \lesssim 6\pi\hbar^3/(Qm_oc)^2$), and to account for the macroscopic world we observe, we assume that $\kappa(m_o|gravitational) \approx \kappa_0$, a constant. $\Rightarrow \kappa_0 > 10^{-30}\ s.m$ which corresponds to c.m d.o.f of an atom/molecule in spatial superposition e.g., in Eq. (10) (note that EM interaction is present only in (i.e., corresponds to) the internal (but not c.m) d.o.f of an atom/molecule). Note that in general, κ may also depend on some other properties like density of EM modes in the vicinity of a quanton as it affects $\bar{\mathcal{T}}_{spo}$ [38, 144]. Further note that we may assume that κ is a universal constant like h, and take its value sufficiently small such that it accounts for the smallest lifetime $(86ys)$ observed so far e.g., $\kappa \sim 10^{-39}s.m$ (assuming that for a proton inside the nucleus, $\lambda_{dB} \sim$ fermi). But such an assumption is unnecessary, as shown above. Moreover, following theoretical argument (based on an hypothetical situation) shows that κ cannot be a universal constant: In the limit $q \to \infty$ and/or $m_o \to \infty$ (with λ_{dB} fixed), we obtain $\bar{\mathcal{T}}_{spo,min,th}(m_o,q|EM) \to 0$. $\Rightarrow \kappa \to 0$ which corresponds to no particle at all. (See supplementary material for details.)

We may obtain an approximate lower bound for $\bar{\tau}$ (and hence κ) as follows: To account for the absence of electrostatic self repulsion of electron in an Hydrogen atom within the quantum field theory (QFT) formulation of NPI, nonlocal hoppings of the electron cannot entail exchange of virtual photons with itself (see supplementary material for details). Hence, EM interaction between electron and proton (and also electron with itself which gives rise to Lamb shift in Hydrogen atom [16, 42, 126]) via exchange of virtual photons [126] should take place within a given interval (t'_i, t'_{i+1}). This is possible provided $\bar{\tau}(e^-|EM) \gtrsim a_0/c \sim as$ where a_0 is the Bohr radius. Note that if $\bar{\tau}(e^-|EM) \ll as$ then there will be no sufficient time for exchange of virtual photons between electron and proton during a given interval (t'_i, t'_{i+1}). $\Rightarrow \kappa(e^-|EM) \gtrsim 10^{-30}s.m$ provided for an electron in an atom/molecule $\lambda_{dB} \gtrsim h/(m_oc)$. Similarly in case of strong interaction we obtain $\bar{\tau}(proton/neutron|strong) \gtrsim fm/v > fm/c \sim 10ys$ where v is the speed of π-mesons (force carriers of strong nuclear force) which have finite rest mass [15]. Hence only within NPI-QFT (but not SQM-QFT) we have a satisfactory physical explanation for the experimentally observed shortest excited state life time of a nucleus $(86ys)$ and also that of an electron $(\sim as)$.

Experimental evidence in favor of $\bar{\tau} = \kappa/\lambda_{dB}$: In Talbot-Lau and Kapitza-Dirac-Talbot-Lau matter-wave interferometers, the total time taken by a molecule/quanton to manifest/exhibit wave-nature/interference is given by the Talbot time $T = ad^2/(\lambda_{dB}v)$ where d is the grating period, v is the magnitude of longitudinal velocity of the molecule, and a is a dimensionless constant [8, 61]. Then for given values of $0 < v < c$ and d, we have $T \propto 1/\lambda_{dB}$ (more precisely $1/\bar{\lambda}_{dB}$ because an eigenstate of momentum is not physical). This justifies and is justified by Eq. (1) which $\Rightarrow T \approx f\bar{\tau}$ where f is the total number of hoppings during T [Time]. Note that the relation $T = ad^2/(\lambda_{dB}v)$ has been verified experimentally for both light [8] and matter [53, 61, 79, 121]. Even though $T = ad^2/(\lambda_{dB}v)$ can be obtained using SE [32], only within NPI (but not standard quantum mechanics (SQM)[42]/BM) we can give a natural and satisfactory physical explanation/reason for the dependency $T \propto 1/\lambda_{dB}$, via Eq. (1). Further if we assume that f (for given v, d) is same for all particles (micro and macro) then a macroscopic particle (with $\bar{\tau} \gg 1$) will not exhibit interference on practical time scales ($\because$ of no hoppings) even if there is no decoherence due to black body radiation [8, 79]. Hence within NPI, there seems to be no need of wave function collapse, many worlds interpretation (MWI), decoherence etc., to account for the macroscopic world we observe. (See supplementary material for details.)

Similarly for a single spin-s particle we define $\bar{\tau}(\text{spin}) = \kappa'\sqrt{s(s+1)}$ where κ' is a proportionality constant. Note that unless specified, $\bar{\tau}$ corresponds to c.m d.o.f of atom considered in the state (2).

Deriving NPI solely from the principle of unification of wave and particle, by eliminating hidden variable:
1. Consequences of wave nature exhibited by a quanton: Position and momentum do not commute and consequently they cannot have definite values simultaneously (uncertainty principle), Bell [84] and AB [4] quantum nonlocality, quantum contextuality [29, 76], randomness in measurement outcomes, tunneling, refraction, diffraction, interference, particle properties like mass, charge etc., are delocalized in position basis (in a square-integrable state e.g., in AB effect), and existence of stable (against radiation from an accelerating charge) states of the electron in an Hydrogen atom.
2. Properties exhibited by a particle with hidden variable which is not coupled to a quantum nonlocal guiding wave (i.e., a classical particle): Both position and momentum will always have definite premeasurement ontological values simultaneously and hence they commute (classical determinism), classical locality, classical noncontextuality, no randomness in measurement outcomes, no tunneling, no refraction, no diffraction, no interference, particle properties like mass, charge etc., are always localized at the hypothetical position of the particle, and stable states of the electron in an Hydrogen atom are not possible [42].

This shows that consequences-1 and properties-2 are inconsistent with each other. These inconsistencies are well known from the experimental violation of Bell-CHSH inequality [84], noncontextual hidden variable inequalities [47, 92], "crypto-nonlocal" hidden variable inequalities [107], Leggett-Garg inequality (even by nuclear spins [10]); Peres-Mermin square [118, 125]; non-conservation of spin magnitude in a hidden variable model [104]; [145] etc. Hence existence of a particle with hidden variable simultaneously with an empty nonlocal guiding wave implies existence of two separate and independent realities/worlds, as their properties always contradict with each other. Hence the principle of unification of wave and particle strictly prohibits the same as they cannot be unified even though they might be coupled (via the guidance equation) in an ad-hoc fashion like in BM (p. 45 [87]) and Bell's hidden variable model of spin-1/2 [118], to achieve consistency with SQM in the postmeasurement scenario even at the level of just two quantons/measurements (this is also justified by the inconsistencies between experiments and BM discussed earlier). Moreover in both BM and Bell's model, randomness in the initial value of hidden variable has to be imposed from outside to achieve consistency with SQM (note that in case of a particle without hidden variable, randomness emerges from within). (See supplementary material for details.) This is further justified by Bell's statement: "For me the possibility of determinism is less compelling than the possibility of having one world instead of two." p. 30 [17]. Further linearity of SE makes sense only if full (but not a fraction of) mass/energy, charge, magnetic dipole moment etc., are spatially-delocalized/present across each of the two split-wave-packets/interfering-paths (e.g., in AB effect, Stern-Gerlach (SG) wave packet splitting [111], gravitationally induced neutron interference [43] etc.). Only then the Hamiltonian can be active/on at both the wave-packets/paths there by making the dynamics linear. Hence it does not make sense to describe the time evolution of an empty wave packet using linear SE (like in BM [81, 152]) as it is devoid of particle properties (this motivated postulate-4).

Conjecture-1: Particle (i.e., wave/field excitation) acquires properties like mass and definite values of both position and momentum simultaneously during (t'_i, t'_{i+1}), by virtue of Higgs field [126] and spacetime together $\forall i$. Note that in QFT, field is an operator which creates particle but even the (plane) wave is built into its definition [126]. But here for convenience/simplicity we separate out the wave from the particle. Conjecture-2: Spacetime and matter themselves emerge from an underlying empty nonlocal guiding wave/field as quanta of field excitations. Note that without conjecture-2, nonlocal hopping is not logical and we cannot satisfactorily account for the experimental fact that we cannot directly observe the empty nonlocal guiding wave, as discussed in postulate-2. Hence within NPI, particle is only apparent (i.e., not real like in BM) as it is just an excitation of an underlying empty wave/field which momentarily acquires particle properties and vanishes completely at the instant t'_i of quanton hopping $\forall i$. Hence within NPI, there is only one reality/world viz., an underlying absolutely empty wave. Hence the principle of unification of wave and particle allows the concept of a particle without hidden variable as defined in postulate-1 which is both necessary and sufficient to eliminate hidden variable. Note that the experiment proposed here (Eq. (10))

to test NPI (postulate-3), will not directly test the above two conjectures (however verification of postulate-3 will indirectly justify/support conjectures-1,2). (See supplementary material for details.)

Deriving NPI starting from a primary field: Assume there exists a primary field/wave which is devoid of all the qualities i.e., particle properties like definite values of position and momentum, mass, charge, spin etc. In other words, assume there exists an underlying absolutely empty (i.e., devoid of all the particle properties) quantum nonlocal field/wave. This primary field is nothing but the Nirguna (quality-less) Brahman of Advaitha (non-dual) Vedanta, the Maha-Shunya/Nirvana (supreme void/vacuum) of Buddhism [146]. From this primary-field emerges matter, energy and spacetime as quanta of field excitations (for a similar idea see [146]). These excitations acquire qualities (particle properties) via interaction with each other. The excitation/particle survives for a time period τ. If $\tau = \infty$ then the particle could not have emerged from the underlying primary field. This is because the concept of emergence/excitation/construction makes sense only if there is merging-back/de-excitation/destruction as well which corresponds to $\tau < \infty$. Moreover $\tau = \infty$ implies the existence of two independent worlds/realities (like in BM) as the properties of nonlocal-wave and that of localized-particle contradict with each other, which in turn contradicts our starting assumption of existence of a single primary field. Hence the excitation/particle can survive only for a finite time period and hence $0 < \tau < \infty$. $\tau > 0$ is necessary for the manifestation of particle nature.

If we assume τ is a finite constant equal to the time period from big-bang to big-crunch of the universe then we need to introduce an ad-hoc artificial external coupling between wave and particle (like in BM) to account for the wave natures exhibited by the particle. Then on practical time scales, the model becomes BM which is not satisfactory and not consistent with all the observed phenomena at the microscopic domain, as discussed earlier. If we assume τ is a small constant such that it accounts for the observations at the microscopic domain (via nonlocal hoppings) then it cannot account for the macroscopic world we observe wherein the objects are not hopping in a nonlocal and random fashion. Further as τ is a measure of particle nature, it is justifiable to conjecture that it depends on λ_{dB}. Hence τ must be a random variable and we postulate that its mean value $\bar{\tau} \propto 1/\lambda_{dB}$ which in turn naturally accounts for the emergence of classical world from an underlying quantum world, like $p = h/\lambda_{dB}$ does. As the underlying wave is nonlocal, there is no necessity of time delay between destruction/merging at $(\vec{r}, t = \tau)$ and reconstruction/reemerging at $(\vec{r}', t')$, and also such a time delay violates conservation of mass, energy etc., and also not necessary for the manifestation of wave nature as the wave always exists. Hence the particle must instantaneously (i.e., $t' = t$) re-emerge (after merging) at an arbitrary point in space allowed by the underlying wave function, in a random fashion, as no point in space is preferred. This constitutes random nonlocal-hoppings of the particle across spatial points allowed by the underlying wave function. Similar random discontinuous hoppings should exist in non-spatial d.o.f (like spin) as well.

Ideas/models similar to NPI: Based on experimental evidences, Shan Gao [67, 68] and Rashkovskiy [129] have also proposed models similar to NPI. Bell's Everett (?) theory (p. 133 [17]) and the "consistent histories" interpretation of quantum mechanics [49, 122, 159] are also similar in spirit to NPI. Harry's discontinuous hop model [57, 58] is also similar to NPI. (See supplementary material for details.)

Reproducing the known experimental results within NPI: Dynamics (N) within NPI (postulate-4) is nonlinear ($\because E(t)$ depends on wave function). However on time scales $\gg \bar{\tau}$, nonlinear effects can be suppressed by tuning the $b(E(t))$'s such that we approximately recover linear Schrödinger evolution U. The physical origin of b's might be the distortion of the Hilbert space (which increases its curvature) caused by discontinuous hoppings. (See Supplementary material for details.)

A gedanken experiment to test NPI: Entanglement-free measurement of a single unknown quantum state: Consider an internally highly-excited single atom/molecule put in spatial superposition of two distinct locations:

$$
\begin{aligned}
&|\eta(t=0)\rangle \\
&= \Big(\cos\frac{\theta}{2}|0\rangle|\psi_{\vec{r}_0,\vec{p}_0}\rangle + \sin\frac{\theta}{2}|1\rangle|\psi_{\vec{r}_1,\vec{p}_1}\rangle\Big)|0\rangle_f^{\otimes M}|M\rangle_n \quad (2)
\end{aligned}
$$

where

$$
\begin{aligned}
\langle\vec{r}|\psi_{\vec{r}_l,\vec{p}_l}\rangle &= \psi_{\vec{r}_l,\vec{p}_l}(\vec{r}) = \psi_{x_l,p_{lx}}(x)\psi_{y_l,p_{ly}}(y)\psi_{z_l,p_{lz}}(z), \\
\psi_{x_l,p_{lx}}(x) &= (2\pi(\Delta x)^2)^{-1/4} e^{ip_{lx}x/\hbar} \exp(\frac{-(x-x_l)^2}{4(\Delta x)^2}), \\
\langle\vec{p}|\psi_{\vec{r}_l,\vec{p}_l}\rangle &= \bar{\psi}_{\vec{r}_l,\vec{p}_l}(\vec{p}) = \bar{\psi}_{x_l,p_{lx}}(p_x)\bar{\psi}_{y_l,p_{ly}}(p_y)\bar{\psi}_{z_l,p_{lz}}(p_z), \\
\bar{\psi}_{x_l,p_{lx}}(p_x) &= (2\pi(\Delta p_x)^2)^{-1/4} e^{-ip_x x_l/\hbar} \exp(\frac{-(p_x-p_{lx})^2}{4(\Delta p_x)^2}), \quad (3)
\end{aligned}
$$

$\Delta x \Delta p_x = \hbar/2$ [42], $l = 0$(location-0), 1(location-1), $p_{0z} = -p_{1z}, p_{0y} = p_{1y}, p_{0x} = p_{1x} = 0$, and similar expressions hold for wave packets corresponding to y and z d.o.f as well; $\langle\psi_{\vec{r}_i,\vec{p}_i}|\psi_{\vec{r}_j,\vec{p}_j}\rangle = \delta_{ij}, i,j = 0,1$; $\vec{R}|\vec{r}\rangle = \vec{r}|\vec{r}\rangle, \vec{P}|\vec{p}\rangle = \vec{p}|\vec{p}\rangle$ are the position and momentum operator relations (eigenvalue equations) respectively, corresponding to c.m d.o.f of atom; $|0\rangle(|1\rangle)$ is an eigenstate of the z-component of atom's nuclear/electronic spin angular momentum operator S_z with eigenvalue $+\hbar/2(-\hbar/2)$; $|M\rangle_n$ is the M-th excited nuclear/electronic energy eigenstate of the atom; and $|0\rangle_f$ is the ground state of quantized EM field (which is an independent d.o.f) i.e., quantized vacuum. $|M\rangle_n$ couples to the quantized vacuum and emits M photons spontaneously from location-i (provided the atom is at location-i) in a cascaded fashion via intermediate metastable states [5, 9, 154], $i = 0, 1$. As each of the M photons emitted from location-i are distinguishable via their time of arrival, in Eq. (2) we have associated an independent quantized vacuum state with each of them, $i = 0, 1$. This is further justified by the fact that each of the M spontaneous emission events are independent. Consequently no need to symmetrize the wave function. Spatial superposition in (2) can be achieved e.g., using SG technique [111] (also see [2, 42, 82]), light pulse [21, 91], mechanical[36, 79, 97]/optical[61, 128] matter-wave diffraction grating, or trapping in a double well potential [138] (last two do not require spin/internal d.o.f to split the c.m wave packet). Note that the d.o.f corresponding to nuclear energy eigenstates, electronic energy eigenstates, and c.m of the entire atom/molecule are different and independent of each other [8]. First two correspond to the dynamics with respect to c.m and the last one corresponds to motion of the c.m [42].

Let us assume that Alice has prepared the state (2), and θ is unknown to Bob. We are going to show that within NPI, Bob can deterministically measure θ using a single copy of the unknown state (2), with good precision. For the sake of conceptual clarity let us assume that the life time of excited state $|1\rangle$ is much greater than that of $|M\rangle_n$ and consequently we neglect the time evolution of spin d.o.f in the following calculations. And assume that coupling between atom and quantized EM field is turned on only after preparing the state (2) [111] or no radiative decay until (2) is prepared.

The interaction Hamiltonian H_I^j between c.m and internal d.o.f of atom and quantized EM field is given by [77]:

$$\begin{aligned} H_I^j &= \int d^3p' \\ \times\big(\vec{\mu}_{M-j,M+1-j}.\vec{g}_{\vec{p}'} e^{-\frac{i}{\hbar}\vec{p}'.\vec{R}} a^\dagger_{\vec{p}'} &|M-j\rangle\langle M+1-j|_n \\ +\vec{\mu}_{M+1-j,M-j}.\vec{g}^*_{\vec{p}'} e^{\frac{i}{\hbar}\vec{p}'.\vec{R}} a_{\vec{p}'} &|M+1-j\rangle\langle M-j|_n\big), \end{aligned} \tag{4}$$

$$\begin{aligned} H_o^j &= \hbar\omega_{M+1-j}|M+1-j\rangle\langle M+1-j|_n \\ &+\hbar\omega_{M-j}|M-j\rangle\langle M-j|_n + \int d^3p' \hbar\omega_{\vec{p}'} a^\dagger_{\vec{p}'} a_{\vec{p}'} \end{aligned} \tag{5}$$

where $\vec{\mu}_{M+1-j,M-j}$ is the matrix element of atomic electric dipole moment operator $\vec{\mu}$ between the states $|M+1-j\rangle_n, |M-j\rangle_n$; $\omega_{\vec{p}'}$ and $\vec{g}_{\vec{p}'} = i\sqrt{2\pi\hbar\omega^2_{\vec{p}'_{oj}}/(L^3\omega_{\vec{p}'})}\vec{\epsilon}_{\vec{p}'}$ are the frequency and amplitude respectively of the field corresponding to the mode $\vec{p}', |\vec{p}'| = \hbar\omega_{\vec{p}'}/c$; L^3 is the quantization volume; d^3p' is the differential volume element; $\hbar\omega_{M+1-j}$ is the energy corresponding to the state $|M+1-j\rangle_n$; $\omega_{\vec{p}'_{oj}} = \omega_{M+1-j} - \omega_{M-j}$; $\vec{\epsilon}_{\vec{p}'}$ is the field polarization unit vector; $a_{\vec{p}'}(a^\dagger_{\vec{p}'})$ is the annihilation (creation) operator corresponding to the field mode $\vec{p}'$ i.e., $a_{\vec{p}'}|\vec{p}'\rangle_f = |0\rangle_f$ $(a^\dagger_{\vec{p}'}|0\rangle_f = |\vec{p}'\rangle_f)$, $a_{\vec{p}'}|0\rangle_f = 0$; and in Eq. (4) we have used rotating wave approximation and the total Hamiltonian (neglecting the zero-point energy of the quantized vacuum) $H_{U_j} = \frac{\vec{P}^2}{2m} + H_o^j + H_I^j$ corresponds to nonrelativistic limit, m is the mass of the atom; $j = 1, 2, ..., M$.

Linear Schrödinger evolution: By virtue of linearity of SE, the two superposed states in (2) evolve without coupling/interacting, as follows. Let $U = U_M...U_j...U_2U_1$ where U_j is the linear Schrödinger time evolution operator corresponding to H_{U_j} which induces spontaneous emission of j-th independent photon. We make the following assumptions: 1. Both $\Delta x, \Delta p_x$ are small such that the wave packet $|\psi_{\vec{r}_l,\vec{p}_l}\rangle$ is sufficiently localized around x_l, p_{lx} in position, momentum basis respectively, $l = 0, 1$. And similar assumptions hold for wave packets corresponding to y, z d.o.f as well. This in turn requires spreading of the wave packet in position basis should be negligible during the time scales of interest. 2. Counting time is much greater than the atomic decay time (p. 208 [156],[59]) (which is equivalent to taking the limit $t \to \infty$ during which the probability of internal d.o.f remaining in the excited state $|M\rangle_n$ goes to zero exponentially [6, 42, 77]). Then by virtue of assumption-1 we can make the approximation that $|\psi_{\vec{r}_l,\vec{p}_l}\rangle$ is an approximate simultaneous eigenstate of both position and momentum with eigenvalues $\vec{r}_l, \vec{p}_l$ respectively which in turn leads to loss of entanglement between atom and the photon it emits (this can be shown via Green's function) $l = 0, 1$. This is justified by the fact that uncertainty in position and momentum of the atom in the state $|\eta(0)\rangle$ are much greater than those in the state $|\psi_{\vec{r}_l,\vec{p}_l}\rangle, l = 0, 1$. Then using Green's function (propagator for the SE [42]) [77] we obtain:

$$\begin{aligned} |\eta(t)\rangle_U &= U|\eta(t=0)\rangle \\ \approx \Big(\cos\frac{\theta}{2}|0\rangle|\psi_{\vec{r}_0,\vec{p}_0}(t)\rangle &\bigotimes_{j=1}^{M} |\phi_{\vec{r}_0,\vec{p}'_{oj}}(t)\rangle_f \\ +\sin\frac{\theta}{2}|1\rangle|\psi_{\vec{r}_1,\vec{p}_1}(t)\rangle &\bigotimes_{j=1}^{M} |\phi_{\vec{r}_1,\vec{p}'_{oj}}(t)\rangle_f\Big)|0\rangle_n \end{aligned} \tag{6}$$

(see supplementary material for details) where

$${}_f\langle\vec{p}'|\phi_{\vec{r}_l,\vec{p}'_{oj}}(t)\rangle_f = \frac{A\sin\theta_{\vec{p}'}}{\sqrt{p'}} \frac{e^{-\frac{i}{\hbar}\vec{p}'.\vec{r}_l} e^{-\frac{i}{\hbar}p'ct}}{p' - p'_{oj} + i\Delta p'/2}, \tag{7}$$

$$\begin{aligned} &{}_f\langle\vec{r}'|\phi_{\vec{r}_l,\vec{p}'_{oj}}(t)\rangle_f \\ \approx \frac{B\sin\theta_{\vec{r}''}}{r''}&\theta(ct-r'')e^{-\frac{i}{\hbar}p'_{oj}(ct-r'')}e^{-\Delta p'(ct-r'')/(2\hbar)} \end{aligned} \tag{8}$$

are the wave functions of the photon in momentum and position bases respectively at time $t \geq t_j$ [59, 134] (note that the wave packet (8) is created at time $t = t_j < t_{j+1}$ where t_j is that instant of time when the energy corresponding to j-th photon is irreversibly lost to the EM field from the atom, $j = 1, 2, ..., M$); A, B are normalization constants; $p' = |\vec{p}'|, \vec{r}'' = \vec{r}' - \vec{r}_l$; $\theta(ct - r'')$ is the Heaviside unit step function [7] used to describe photon's wave function with respect to a photon detector located at $\vec{r}'$ [59]; $\theta_{\vec{q}}$ is the angle between $\vec{q}$ and $\vec{\mu}$, $\vec{q} = \vec{p}', \vec{r}''$ [134]; $\Delta p'$ is the uncertainty in radial component of momentum of photon i.e., the full width at half height (FWHH) of $|_f\langle\vec{p}'|\phi_{\vec{r}_l,\vec{p}'_{oj}}(t)\rangle_f|^2 p'\Delta p'/(2|A|^2 \sin^2\theta_{\vec{p}'}), l = 0, 1\forall j$ [33]; and $_f\langle\vec{r}'|\vec{p}'\rangle_f = e^{i\vec{p}'.\vec{r}'/\hbar}$ [126]. Wave function (7) follows from the Wigner-Weisskopf theory of spontaneous emission which predicts a Lorentzian line shape for the photon [6, 59, 156]. Wave function (8) follows from (7) via Fourier transformation under the following approximations [59]: Far zone approximation i.e., $p'r'' \gg \hbar$ (this follows from the earlier assumption-2 i.e., $t \gg \hbar/(c\Delta p')$). This in turn implies that we can neglect the incoming spherical wave $\exp(-ip'r''/\hbar)$ as its contribution to the probability amplitude will be negligible. Under the far zone approximation we can consider $\vec{p}'$ as nearly parallel to $\vec{r}'' \Rightarrow \theta_{\vec{p}'} \approx \theta_{\vec{r}''}\forall\vec{p}'$. And integrate p' from $-\infty$ to ∞ and use method of residues after extending p' to the complex plane. Note that in (8) if we fix t and assume that $0 < r_l \ll 1$, then upon decreasing r' from its maximum value (which corresponds to $r'' = ct$) to 0, $|_f\langle\vec{r}'|\phi_{\vec{r}_l,\vec{p}'_{oj}}(t)\rangle_f|^2$ also decreases almost exponentially [59] which in turn implies a wave packet localized around the point corresponding to $r'' = ct$, and the corresponding FWHH is given approximately by $\hbar/\Delta p', l = 0, 1\forall j$ [59]. Further as the wave function (8) vanishes for $r'' > ct$, one can normalize the same for any given $t < \infty$. $_f\langle\phi_{\vec{r}_i,\vec{p}'_{oj}}(t)|\phi_{\vec{r}_k,\vec{p}'_{oj}}(t)\rangle_f = \delta_{ik}, i, k = 0, 1\forall j$ because we can spatially resolve the photons coming from the two locations 0 and 1. Further note that in Eqs. (6,7) there is an independent quantum harmonic oscillator (QHO) corresponding to each mode $\vec{p}'$, and by $|\vec{p}'\rangle_f$ we mean, only the QHO corresponding to the mode $\vec{p}'$ is excited and all others are in the ground state. And in Eq. (2), $|0\rangle_f$ represents the combined ground state of all QHOs. (See supplementary material for details.)

Let $\theta = 0$. Then in the exact state $|\eta(t)\rangle_U$ (wherein we do not make the approximation that both position and momentum of the atom have definite values simultaneously), there will be tiny entanglement between atom and photon unlike in the state (6) (however note that in the atom-photon relative-motion coordinate system, we can still express the global atom-photon wave function in an entanglement free form, using far-zone approximation [59]; but if $m \to \infty$ and if atom is in an eigenstate of position then there will be no entanglement between atom and photon even in the lab frame [59]; if the photon is emitted in an eigenstate of momentum then also no entanglement between c.m d.o.f of atom and photon [103]; when an electron[110]/molecule[22] interacts with a diatomic[110]/nano-mechanical[22] matter-wave diffraction grating (whose slits are coupled [110]), there will be no significant amount of entanglement between electron/molecule and the grating even though there is recoil (otherwise it would have degraded the high contrast interference fringes, at least to some extent) [22] (note that this may not hold in case of decoupled slits [110]); even in the protective measurement, entanglement between the quantum system being measured and the measuring device is zero in the limit of extreme adiabaticity [82]; these observations show that momentum-conservation/recoil do not necessarily imply entanglement between atom and the photon it emits; we are going to show that within NPI, even when the photon is not emitted in an eigenstate of momentum, still one can avoid entanglement between c.m d.o.f of an atom with finite mass and the photon it emits). This tiny entanglement persists even when $\theta > 0$ in the exact state $|\eta(t)\rangle_U$ but atom and photon will be still perfectly correlated (unlike m_u photons in $|\eta(t)\rangle_N$ (Eq. (10))) provided assumption-1 is satisfied.

Paradoxes within SQM: It is an experimental fact that an electron in an Hydrogen atom has no electrostatic self-repulsion even though it is spatially delocalized [42, 67, 68, 135, 159]. In SQM (and also in QFT [126]), this is mathematically taken into account via the linearity of SE [68]. Even the experimental fact that classical EM waves in vacuum have neither EM self-interaction nor interact with each other electromagnetically, is taken into account mathematically via the linearity of Maxwell's equations (ME) [88]. As classical EM waves possess no electric charge and exhibit no particle nature, we encounter no paradoxes. But as an electron has both charge and also exhibits particle nature, we encounter following two paradoxes from physics point of view: 1. If we associate an independent particle with each state in spatial superposition or assume that the same single particle/electron itself exists at all the points (allowed by the wave function) in position basis, simultaneously and always (like in some interpretations of Feynman's path integral formulation of SQM wherein a single particle is assumed to explore all possible paths simultaneously or like in MWI) then it leads to the violation of conservation of mass/energy (see p. 1263 [46]), charge etc., in the premeasurement scenario, and also it contradicts the very definition of a particle within NPI. 2. If we assume that mass, charge etc., are statically delocalized across the entire matter-wave like in some interpretations of BM [23] then there can be no particle at all (but only pure wave exists) which fails to account for the particle behaviors exhibited by a quanton discussed earlier. In both of the preceding paradoxes/cases 1 and 2 (except MWI), it is physically unjustifiable and unappealing that the spatially delocalized electron in an Hydrogen atom do not repel itself [67] even though mathematically (i.e., by virtue of linearity of SE) the same is justifiable. Also, case-2 contradicts the very short range nature of strong nuclear force (in case of a spatially delocalized neutron[43, 108]/nucleus[91, 99, 127]), not consistent with the linearity of SE which demands full (but not a fraction of) mass, charge etc., to be associated with each state in spatial superposition, and violates quantization of charge. Note that MWI is not satisfactory and not testable (see supplementary material for details).

Hence, only dynamic delocalization of mass, charge etc., within NPI, can satisfactorily explain (from physics point of view) the absence of electrostatic self-repulsion of electron in an Hydrogen atom [67, 68] (note that this can be explained even within the QFT formulation of NPI, as discussed above) and also the very short range nature of strong nuclear force. Further if we assume nonexistence of hidden variable (which is demanded by unification of wave and particle) then the deterministic nature of linear Schrödinger evolution itself implies that something is missing in it (note that N is stochastic). This shows that SQM (and also QFT) is only a phenomenological model (i.e., a mere set of mathematical tools developed to explain certain experimental results) which do not tell anything about the nature of particle i.e., with or without hidden variable, and also wave i.e., if it is empty or not, if wave is physically real or not, its nonlocal features etc. Note that SQM is based on $p = h/\lambda$ [42] which holds for both light and matter, and consequently SQM considers both wave (λ) and particle (p) natures of a quanton (latter is evident from the absence of electrostatic self-repulsion (of electron) term in the SE of Hydrogen atom [65, 135] and also from the fact that SQM takes into account recoil via entanglement [77]). Above paradoxes within SQM can be satisfactorily resolved only within NPI.

Measurement within SQM: Upon measuring the position of the spontaneously emitted photons in (6) (e.g., we measure in the orthonormal basis $\{|\phi_{\vec{r}_0,\vec{p}'_{oj}}(t)\rangle_f, |\phi_{\vec{r}_1,\vec{p}'_{oj}}(t)\rangle_f\}$ corresponding to a given j), due to wave function collapse, one of the two states/wave-packets in superposition simply disappears without leaving back any footprints (note that if we associate an independent particle with each wave packet in superposition then simply disappearing of photons, violates second law of thermodynamics as spontaneous emission is irreversible in practice [6, 19, 42, 163] and as spontaneous emission happens with probability one), and all the M photons will come from either location-0 or location-1 i.e.,

$$|i\rangle|\psi_{\vec{r}_i,\vec{p}_i}(t)\rangle \bigotimes_{j=1}^{M} |\phi_{\vec{r}_i,\vec{p}'_{oj}}(t)\rangle_f |0\rangle_n \qquad (9)$$

where $i = 0(1)$ with probability $\cos^2(\theta/2)(\sin^2(\theta/2))$. Note that if we assume macroscopic superposition (at least for a short time) then the above paradoxes within SQM are greatly exaggerated in the Schrödinger cat paradox.

Nonlinear evolution within NPI: Using Green's function (for EM field and atom's internal d.o.f only) and assumptions 1-4 (3 and 4 are mentioned below), $|\eta(t=0)\rangle$ evolves under NPI's nonlinear evolution operator N as follows,

$$\begin{aligned} &|\eta(t)\rangle_N = N|\eta(t=0)\rangle \\ &\approx \Big(\cos\frac{\theta}{2}|0\rangle|\psi_{\vec{r}_0,\vec{p}_0-\sum_{j=1}^{m_u}\vec{p}'_{oj}}(t)\rangle \bigotimes_{k=1}^{m_e} |\phi_{\vec{r}_0,\vec{p}'_{ok}}(t)\rangle_f \\ &+ \sin\frac{\theta}{2}|1\rangle|\psi_{\vec{r}_1,\vec{p}_1-\sum_{j=1}^{m_u}\vec{p}'_{oj}}(t)\rangle \bigotimes_{k=1}^{m_e} |\phi_{\vec{r}_1,\vec{p}'_{ok}}(t)\rangle_f \Big) \\ &\bigotimes_{j=1}^{m_u} |\phi_{\vec{r}_w,\vec{p}'_{oj}}(t)\rangle_f |0\rangle_n \qquad (10) \end{aligned}$$

where w is an independent random variable which outputs $0, 1$ with probability $\cos^2(\theta/2), \sin^2(\theta/2)$ respectively $\forall j$, and $m_e + m_u = M$. Note that the probability distribution of w is independent of $\bar{\tau}, \bar{\mathcal{T}}_{spo}$ but only m_u depends on $\bar{\tau}, \bar{\mathcal{T}}_{spo}$ where $\bar{\mathcal{T}}_{spo}$ is the average time taken by a spontaneous photon emission event. There is no entanglement between atom and m_u photons by virtue of particle (corresponding to c.m d.o.f of atom) having definite values of both position ($\vec{r}$) and momentum ($\vec{p}$) simultaneously during each of the m_u independent spontaneous emission events (postulate-3). Consider $m_u = M = 1$. Then there are no hoppings in c.m d.o.f of atom during the time period of interest and consequently only particle nature is manifest (postulate-3; note that here scalar and vector potentials are zero). And hence with respect to manifest part, the state of the atom (c.m d.o.f) may be described by a point in the phase-space i.e., $(\vec{r}, \vec{p})$ (we may safely neglect the empty guiding wave). Consequently there can be no entanglement (which is a consequence of wave nature) between atom and the photon it emits. This can be further justified by the calculations within SQM (Eq. (6) with $\theta = 0$) wherein because of the approximation that atom's wave-packet (c.m d.o.f) is an approximate simultaneous eigenstate of both position and momentum, amount of entanglement between atom and photon becomes negligible (this holds even if the potential energy is nonzero). This can also be justified by the fact that when a classical particle disintegrates into two parts, there can be no entanglement between them. When $M > 1$, as spontaneous emission events are independent, preceding arguments hold for each spontaneous emission event and consequently there is possibility of getting $m_u > 1$. NPI SE (Eq. (11)) fails to capture recoil because of no entanglement between atom and m_u photons (this is expected as we are neglecting the wave function corresponding to c.m d.o.f during each of the m_u spontaneous emission events, as prescribed by postulate-3). We calculate the same using Newton's equation and input (the approximate value) into the wave function as done in the state (10). Note that entanglement (linear superposition) is a consequence of linearity of SE [28]. Hence N should be nonlinear as it avoids entanglement between atom and m_u photons. (See supplementary material for details.)

Note that even in the exact state $|\eta(t)\rangle_N$ wherein we make no approximations like on the right hand side of (10), entanglement between atom and $m_u[m_e]$ number of photons will be absolutely zero [slightly more than that in the state (10)]. Hence the approximations made in (10) are with respect to entanglement between atom and m_e photons, and the amount of recoil only. Note that as there is no preferred direction for spontaneous emission, the net effect of recoil will be approximately zero provided $M \gg 1$ and all photons are of equal energy. (See supplementary material for details.) Further note that the approximations used in obtaining expression (8) from (7) cannot affect entanglement between atom and photons in (6, 10). Note that interpretation of the state (6) within NPI is the following: Atom and photons hop together between the two locations 0 and 1 in a correlated fashion by exchanging energy and/or momentum with quantum potential. With this interpretation, the paradoxes within SQM discussed above disappear. Further note that in Eq. (10) we consider the unknown states path by path, independently [101, 102].

We have made the following assumptions-3,4 in obtaining the state (10): 3. A given spontaneous emission event is said to be complete after say e.g., $0.9\bar{\mathcal{T}}_{spo}$ [Time] (from the beginning of the event) such that energy/photon is irreversibly lost to the quantized EM field from the internal d.o.f of atom (i.e., $|M\rangle_n$). And each of the M spontaneous emission events are independent. This implies that, if the atom do not hop in c.m d.o.f during $0.9\bar{\mathcal{T}}_{spo}$ [Time] then the corresponding photon completely escapes entanglement (or it is too weak) with c.m d.o.f of atom and consequently we obtain one of the m_u unentangled photons as described in (10). 4. Even a single discontinuous hopping in c.m d.o.f of atom during $0.9\bar{\mathcal{T}}_{spo}$ [Time] will generate entanglement between atom and photon. Then via entanglement equilibration (over time scales $\gg \bar{\tau}$), we obtain one of the m_e photons given in (10). (See supplementary material for details.)

We have following four possibilities over time scales $\gg \bar{\tau}$: 1. $\bar{\mathcal{T}}_{spo} \ll \bar{\tau}$: In this case we obtain $m_u \gg m_e$. 2. $\bar{\mathcal{T}}_{spo} \gg \bar{\tau}$: Here $m_u \approx 0$. Consequently $|\eta(t)\rangle_N \approx |\eta(t)\rangle_U$. $\Rightarrow$ SQM is only an approximate special case of NPI. 3. $\mathcal{T}_{spo} = \tau$. $\Rightarrow$ a spontaneous emission event happens during $(t'_i, t'_{i+1})\forall i$ i.e., spontaneous emission events are perfectly resonant/synchronized/correlated with hoppings in c.m d.o.f of atom. In this case $m_e = 0$. Note that both $\tau, \mathcal{T}_{spo}$ are random variables whose mean values can be engineered. But as they are independent random variables, it is impossible to achieve perfect resonance condition deterministically, even in principle. However there is nonzero probability of perfect resonance happening naturally (it is more likely to happen if $\bar{\mathcal{T}}_{spo} \lesssim \bar{\tau}$). 4. $\bar{\mathcal{T}}_{spo} \approx \bar{\tau}$ and both hopping and spontaneous emission events start at approximately the same time: Here $m_e \approx 0$ if we are close to resonance condition discussed above else $m_e \sim m_u$.

As both $\bar{\mathcal{T}}_{spo}$ and $\bar{\tau}$ can be engineered, it should be possible to achieve $\bar{\mathcal{T}}_{spo} \lesssim \bar{\tau}$ which in turn may require $\bar{\tau}(\text{photon}) \ll \bar{\tau}$. This is because the phenomena of spontaneous emission requires/involves superposition ($\because \bar{\mathcal{T}}_{spo}$ but not $\bar{\tau}(\text{photon})$ depends on density of EM modes [38, 142, 143], cavity-resonance condition [90], symmetry p. 1269 [42], and transition probability (selection rules) [42]) which in turn may require several hoppings in photon's c.m d.o.f (to manifest wave nature) within $\bar{\tau}$ seconds. But the fact that $\bar{\mathcal{T}}_{spo}$ depends on density of EM modes, cavity-resonance condition etc., implies that achieving $\bar{\mathcal{T}}_{spo} \lesssim \bar{\tau}$ does not depend solely on $\bar{\tau}(\text{photon})$.

An indirect test of NPI at $M = 1$: In the state (10), let $M = 1$ and $\bar{\mathcal{T}}_{spo} < \bar{\tau}$ ($\bar{\mathcal{T}}_{spo} \ll \bar{\tau}$ is preferred). In this case atom and photon will not be entangled (with high probability) unlike in the state (6). Hence we can look for the absence of correlation between atom and photon (by measuring them independently) even at $M = 1$, contrary to the predictions of SQM (Eq. (9)) and BM. Note that lesser the $\bar{\mathcal{T}}_{spo}$ (compared to $\bar{\tau}$), more will be the chances of observing absence of correlation between atom and photon. Further note that we can look for the absence of correlation even with an ensemble of atoms wherein each atom is prepared in the state $|\eta(0)\rangle$ with $M = 1$. And we can also look for the absence of correlation (at $M = 1$) via scattering. (See supplementary material for details.) It is an indirect test because we cannot directly observe nonlocal hopping (via photons coming from both locations 0 and 1) like in the $M > 1$ case.

Measurement within NPI: Within NPI, we may account for the macroscopic world we observe (and hence the unique measurement outcome we obtain upon measurement) just by taking $\bar{\tau}$ (corresponding to the combined entangled atom and measuring device) sufficiently large such that there are no hoppings on practical time scales. Hence there seems to be no need of objective/subjective wave function collapse, MWI etc., to account for the same. This is justified by the fact that when $\bar{\tau} \gg 1$, NPI $\approx$ BM (as shown earlier). And we know that within BM there is no need of wave function collapse to account for the macroscopic world we observe [87, 152]. (See supplementary material for details.)

We are going to measure the position of the spontaneously emitted photons in (10), only after all the M photons have been emitted (this is not necessary if $m_e = 0$). This requires e.g., to place the detector/camera sufficiently far from the atom such that the first photon cannot reach it until the last photon is emitted. Note that if $m_e > 0$ and if we detect the photon(s) even before all the M photons have been emitted, then hoppings (in c.m d.o.f of atom) may stop (on practical time scales) due to entanglement of atom with the measuring device

(induced by photon) which in turn makes the combined $\bar{\tau}$ of atom and measuring device $\gg 1$. Consequently we may not get photons from both locations.
In the case where $m_e \ll m_u$ and $M \gg 1$, intensity of photons coming from location-0 is given by $m_0/M \approx \cos^2(\theta/2)$ where m_0 is the number of photons coming from location-0. $\Rightarrow \cos(\theta/2) \approx \sqrt{m_0/M}(\because 0 \leq \theta/2 \leq \pi/2)$. This shows that deviating from purely wave nature (i.e., $\bar{\tau} = 0$) inevitably entails nonlinear evolution, in general (this motivates postulate-4 wherein the dynamics is nonlinear and linearity (i.e., U) is recovered by tuning b's). And it can be exploited to measure a single unknown quantum state. Note that no-cloning principle is based on linear SE [26, 161].

Dynamical equations governing N: In H_I^j (Eq. (4)), let us replace the operator $\vec{R}$ by its eigenvalue $\vec{r}$ i.e., position of the nonlocal-hopping particle, to obtain $H_I^{j'}(\vec{r}(t))\forall j$. Let $N = N_M ... N_j ... N_2 N_1$ where $N_j(\vec{r}(t))$ is the nonlinear operator which induces spontaneous emission of j-th photon. If we neglect recoil due to spontaneous emission then in those independent spontaneous emission events in which there will be no entanglement between atom and photons, $|\eta(t)\rangle_N, t_{j-1} \leq t \leq t_j$ can be obtained by the solution of coupled NPI SE

$$i\hbar\frac{d}{dt}|\eta(t)\rangle_N = H_{N_j}(\vec{r}(t))|\eta(t)\rangle_N \tag{11}$$

and the master equation (13) where $H_{N_j}(\vec{r}(t)) = b(\vec{p}(t))\frac{1}{2m}\vec{P}^2 + b(E_{f,n}(t))(H_o^j + H_I^{j'}(\vec{r}(t)))$, $E_{f,n}(t)$ is the definite value of energy of the combined-particle corresponding to combined EM field and atom's internal d.o.f, $j = 1, 2, ..., m_u; t_0 = 0$. (Note that we obtain (10) after eliminating/tuning-out b's.) $\Rightarrow N(\vec{r}(t))$ is unitary and hence it preserves the norm. Also, as evident from Eq. (10), $N(\vec{r}(t))$ preserves superposition (i.e., do not induce wave function collapse) even though it is nonlinear (see Eq. (15)) like in the case of a qubit traversing a closed time-like curve (CTC) [24] which allows cloning non-orthogonal quantum states [25]. However $N(\vec{r}(t))$ is irreversible as it is stochastic [87]. Eq. (11) is further justified by the following observation: Energy is stored in the particle ($|M\rangle_n$) which is hopping. Hence Hamiltonian depends on the position of the particle. Note that even in the SG experiment [42], Hamiltonian depends on the position of the particle (via gradient magnetic field) but it is linear i.e., do not depend on the wave function unlike H_{N_j}. Further note that $H_{N_j}(\vec{r}(t))$ do not depend explicitly on t and hence the system is conservative/closed, as required. And because $H_{N_j}(\vec{r}(t))\forall j$ changes discontinuously as the particle hops, the trajectory of $|\eta(t)\rangle_N$ in the Hilbert space may not be smooth even though continuous (see supplementary material for details).

Sample path of the hopping particle in position basis is given by,

$$\vec{r}(t) = \vec{r}(t_i'^{+}) + \frac{1}{m}\int_{t_i'^{+}}^{t} dt'\ \vec{p}(t'), t_i' < t < t_{i+1}'\forall i \tag{12}$$

where $\vec{p}$ is the momentum of particle p. 102 [69]. Note that in Eq. (12) there is no diffusion/noise term (Wiener process) but only drifts and jumps. Hence $\vec{r}(t)$ is continuous and smooth in $(t_i', t_{i+1}')\forall i$. Discontinuous jumps in position basis at $t = t_i'$ i.e., $\vec{r}(t_i')$, are governed by the master equation (pp. 50, 52 [69]),

$$\frac{\partial}{\partial t}P(\vec{r}, t|\vec{r}', t') = \lim_{\varepsilon\to 0}\int_{|\vec{r}-\vec{r}''|>\varepsilon>0} d^3r'' \\ \times\big(W(\vec{r}|\vec{r}'', t)P(\vec{r}'', t|\vec{r}', t') - W(\vec{r}''|\vec{r}, t)P(\vec{r}, t|\vec{r}', t')\big) \tag{13}$$

where $P(\vec{r}, t|\vec{r}', t')$ is the conditional probability density of particle being at $\vec{r}$ at time t given that it was at $\vec{r}'$ at time $t' < t$, and

$$W(\vec{r}|\vec{r}'', t) = \lim_{\Delta t\to 0} P(\vec{r}, t + \Delta t|\vec{r}'', t)/\Delta t \tag{14}$$

is the transition probability density per unit time, $|\vec{r} - \vec{r}''| \geq \varepsilon > 0$ p. 47 [69]. Note that in (14), particle was at $\vec{r}''$ at time t^- and particle will be at $\vec{r}$ at time t^+. This is because at t (the instant of hopping) quanton has no definite value of position and hence we cannot talk of particle (actually it has vanished from position and also momentum bases). To first order in Δt, solution of Eq. (13) is given by,

$$P(\vec{r}, t + \Delta t|\vec{r}', t) = \delta(\vec{r}' - \vec{r})\big(1 \\ - \lim_{\varepsilon\to 0}\int_{|\vec{r}'-\vec{r}''|>\varepsilon>0} d^3r'' W(\vec{r}''|\vec{r}', t)\Delta t\big) + W(\vec{r}|\vec{r}', t)\Delta t \tag{15}$$

where we used the consistency/initial condition $P(\vec{r}, t|\vec{r}', t) = \delta(\vec{r}' - \vec{r})$ p. 52 [69]. In the region $|\vec{r} - \vec{r}'| \geq \varepsilon > 0$, let us define $P(\vec{r}, t|\vec{r}', t') = \alpha|\langle\vec{r}|\eta(t)\rangle_N|^2|t - t'|$ where α is a normalization constant with dimension s^{-1}; and in the inner product, appropriate states corresponding to EM field and atom's internal d.o.f are assumed to be present. $\Rightarrow W(\vec{r}|\vec{r}', t) = \alpha|\langle\vec{r}|\eta(t)\rangle_N|^2$. Hence $N(\vec{r})$ is nonlinear as it depends on $|\eta(t)\rangle_N$. In Eq. (15), coefficient of $\delta(\vec{r}' - \vec{r})$ is the probability of particle staying back at $\vec{r}'$ at $t = t_i'$, and $W(\vec{r}|\vec{r}', t)\Delta t$ is the probability density of particle jumping from $\vec{r}'$ to $\vec{r}$ at $t = t_i'$. Note that τ (and hence t_i') is a random variable. Similar equations hold for discontinuous jumps in momentum basis at $t = t_i'$ i.e., $\vec{p}(t_i')$, as well.

A feasible experiment to test NPI: Schematic diagram of a feasible experimental setup to test/falsify NPI (more precisely, to falsify the possibility of getting photons from both locations 0 and 1) is shown in Fig. 2. It is just a slight modification of the experiments [79, 97] which have been already performed to study the effect of scattering/emission of photons by atoms/molecules (in spatial superposition) on interference. Hence most of the experimental details necessary to test NPI may be directly taken from [79, 97] (and also from [36, 37, 54, 60, 80, 96, 124, 128, 136, 147]) but one has to make sure that there is a single atom/molecule during any given scattering/emission event in the photon detection region (otherwise, even within SQM, photons can come from both locations 0 and 1).

As we do not know a priori the value of κ_0, we should take an atom/molecule as massive as possible (e.g., $25kDa$ [61]) and make/having $\bar{\mathcal{T}}_{spo}$ as small as possible (e.g., excited electronic states have lifetimes from ps to as [14, 38, 100, 113, 148, 150, 151], and excited nuclear states have lifetimes from fs to ys [98, 115, 120, 131]). Hence we may consider vacuum chamber being a resonant cavity[90]/plasmonic-cavity [38] and/or made of material which increases the density of EM modes in the vicinity of atom/molecule and consequently decreases $\bar{\mathcal{T}}_{spo}$ [142, 143]. However, as discussed earlier, it is reasonable to take $\kappa_0 > 10^{-30}s.m.$ Then we should get photons from both locations 0 and 1 from an atom/molecule satisfying $\bar{\mathcal{T}}_{spo}\lambda_{dB} \sim 10^{-30}s.m.$ If we do not get then it is very likely that, our assumption that if a given spontaneous emission event is completed within a given interval (t'_i, t'_{i+1}) then the state of the atom/molecule corresponding to c.m d.o.f is described by a point in the phase-space and/or Eq. (1) is not correct (but it does not imply that there are no nonlocal-hoppings at all; note that there are already strong experimental evidences in favor of the existence of a nonlocal-hopping particle, as discussed earlier).

The setup in Fig. 2 can be used for both direct ($M > 1$) as well as indirect ($M = 1$) test of NPI with either internally excited atom/molecule or via scattering. In the direct test, the distance D should be such that the first scattered/emitted photon (in the detection region) cannot reach the photo-multiplier tube (PMT) until the last photon is scattered/emitted (this is not necessary if we are looking for the very first scattered/emitted photon being unentangled with the atom/molecule; this is not necessary even in the $M = 1$ indirect test). It is important to note that the spatial separation L between the two wave packets in superposition can be even much less than the wavelength of the photon scattered/emitted by the atom/molecule. This is because we are not detecting the m_u photons scattered/emitted along x-axis (i.e., Poynting vector along x-axis) but only that along z-axis; an m_u photon (of arbitrary wavelength) scattered/emitted from location-0 cannot enter location-1 because of the photon blocker in between the two locations; and we are using two independent PMTs one at each location 0 and 1, and hence we are not interested in the spatial resolution of photons at all. Hence whatever the wavelength of the scattered/emitted photon, there is possibility of both PMT 0 and 1 clicking provided the condition $\bar{\mathcal{T}}_{spo} < \bar{\tau}$ is satisfied or no hoppings occur (in c.m d.o.f of atom/molecule) during at least one spontaneous emission event. And this is sufficient to unambiguously verify NPI. Note that to test NPI, wavelength of the scattered/emitted m_u photons matters only if we use a single camera along x-axis. And hence such a setup/configuration/arrangement is not the best/optimal to test NPI. Further note that instead of using gratings, we may use SG technique [42, 111] or light-pulse [91] to achieve spatial superposition but it may not be as feasible as using gratings because then we require two independent internal d.o.f to split the c.m wave packet and for scattering/spontaneous-emission.

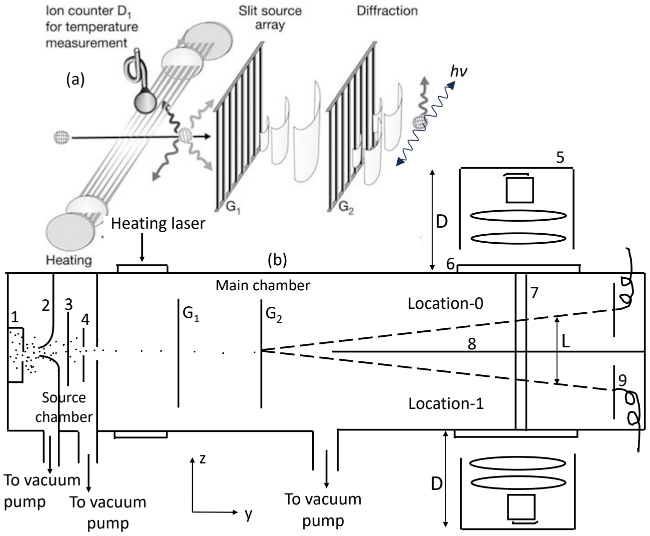

FIG. 2. (a) 3-Dimensional view of a part of the bottom figure (adopted from [79]). (b) Top view of the experimental setup (shown in (a)) to test NPI either by scattering photons off of a single atom/molecule or via a single internally highly excited atom/molecule in spatial superposition: 1. Atomic/molecular furnace/oven [124]. 2. Skimmer [80, 103, 147]. 3. Spinning chopper [60] or Fizeau slotted disks velocity selector [132, 133, 157]. 4. Collimating slit [103, 128]. Note that single atoms/molecules enter the main chamber one after the other such that during more than 99% of the spontaneous photon emission/scattering events, there will be a single atom/molecule in the photon detection region [119]. G_1, G_2 are free-standing (mechanical/material) gratings [79]. 5. PMT and lenses. 6. Window for collecting photons scattered/emitted by atom/molecule along z-axis. 7. Window for probe light-sheet (Poynting vector along negative x-axis), only for testing NPI via scattering. 8. Photon blocker from location 0 ⇄ 1, like the 10 micron thick Copper[54]/Silicon[80] foil physically separating the wave packets in spatial superposition. 9. Mass spectrometer [61]. Long dashed lines represent atom/molecule in spatial superposition.

The interference experiments [35, 36, 52, 79, 97, 128] do not throw any light on NPI for the following reasons even though many photons were scattered[79]/emitted[97] by each atom/molecule in spatial superposition: Their results and conclusions are based on noncommuting observables (i.e., complementarity relation; justified below) whereas to test NPI we use commuting observables (we infer wave nature via nonlocal hopping but not interference). Consequently in their experiment, wavelength of the scattered/emitted photon matters unlike in NPI (as explained above). Their results are based on m_e entangled photons whereas to test NPI we consider/look-for only m_u unentangled photons and hence their arguments do not apply. m_e entangled photons carry path information (because upon detecting them, they project the atom/molecule onto one of the two interfering paths; even if we do not detect them still information will be distributed in the global space due to entanglement which in turn leads to reduction in interference fringe contrast depending on the amount of entanglement) unlike the m_u unentangled photons (note that in the case $m_u = M$, we should obtain full interference fringe contrast provided the reduction in the temporal coherence length due to recoil p. 24 [147] is negligible). Within NPI, we can account for the loss of interference in [79, 97] even by assuming $m_u = M$ but recoil significantly reduced temporal coherence length. Hence there is possibility that many unentangled photons were already emitted in [79, 97]. They did not detect photons scattered/emitted by a single atom/molecule and look for photons coming from both interfering paths (they had used a beam/ensemble of atoms/molecules). Note that in the experiments [79, 97], even though many photons were scattered/emitted by each atom/molecule in spatial superposition, they still observe significant amount of interference but only when the separation between the two interfering paths was much less than the wavelength of the scattered/emitted photon (this corresponds to significantly low entanglement between atom and photon i.e., unsharp/weak measurement). This in turn implies that as wavelength of the scattered/emitted photons do not really matter to test NPI (as shown above), disturbance caused by recoil and entanglement (due to m_e photons) to temporal coherence length and spatial superposition respectively, are not really impediments to test NPI. Hence the optimal/best condition/setup to test NPI is the one in which wavelength of the scattered/emitted photons is maximum possible (note that it cannot be infinite because then there will be no photon at all) with $\bar{\mathcal{T}}_{spo} \lesssim \bar{\tau}$. (See supplementary material for further details.)

A refined complementarity principle within NPI: Within NPI, there is no complementarity with respect to simultaneous existence of both wave and particle because they exist simultaneously during $(t'_i, t'_{i+1})\forall i$. However on time scales $\gg \bar{\tau}$, we can (cannot) observe both wave and particle natures of a quanton using the same given experimental arrangement/setup via commuting [12, 73] (noncommuting [12, 28]) observables, and consequently complementarity relation of SQM [51, 114] do not (do) apply. (Note that measuring noncommuting observables requires different experimental arrangements.) E.g., in the state (10), absence of correlation between atom and m_u number of photons is a consequence of particle nature of atom and getting photons from both locations 0 and 1 is a consequence of wave nature of atom which can be observed via a single measurement outcome containing several photons. In some situations, wave nature is manifest even during (t'_i, t'_{i+1}), as mentioned in postulate-3. Note that within SQM, as evident from the complementarity relation [51, 114], complementarity is with respect to observation of wave and particle natures of a quanton. As it is based (directly or indirectly; see below for justification) on uncertainty principle (which is a consequence of wave nature and which holds in both BM and NPI), it do not really throw any light on the nature of particle i.e., with or without hidden variable, and the manifestation of both wave and particle natures in the same given experimental arrangement on time scales $\gg \bar{\tau}$. Further, within NPI, even though particle is characterized by having definite values of both position and momentum simultaneously during $(t'_i, t'_{i+1})\forall i$, it is not necessary to measure both position and momentum simultaneously to infer/prove the same. But we can infer the same indirectly via photons coming from both locations 0 and 1, as explained above.

Experiments done so far and NPI: The experiments [27, 34, 36, 40, 41, 44, 50, 74, 78, 79, 83, 93, 95, 97, 116, 128, 130, 139, 164] show that complementarity relations [51, 114] are indeed valid. Complementarity relations are based on either uncertainty principle [114] or entanglement/nonlocal-correlation [51]. As shown earlier, both of these are consequences of wave nature of a quanton(s), and hence they should be connected. This is justified by the following observations: [56] claims that only one observable is used in the complementarity relation based on entanglement and hence it has no connection with uncertainty principle. But the definitions of wave (interference fringe visibility) and particle (which path information) natures in [56] use two states $\rho_D^{(i)}$, $U\rho_D^{(i)}U^\dagger$ of the detector. As U actively changes the basis of measurement, two noncommuting observables are involved in the definitions. Hence there is connection with uncertainty principle. This is justified by the results of [12, 51, 114] wherein complementarity relations have been expressed in terms of uncertainty relations (also see [18, 28, 73]). Even in the delayed choice quantum erasure [95], it appears like there is no connection to uncertainty principle. But the very fact that one of the beam splitters erases path information, itself shows that it actively changes the basis of measurement to the one which does not commute with the measurement basis which gives path information. Hence the results and conclusions of the above mentioned experiments are based on noncommuting observables (e.g., they correspond to m_e entangled photons, as explained earlier) with which both NPI (refined complementarity principle) and BM agrees. Whereas to test NPI we need to look for m_u unentangled photons, as explained earlier. Hence the above mentioned experiments were neither designed nor intended to test NPI. This shows that if we can define/interpret wave and particle behaviors using commuting observables like in [12, 73], then complementarity relation do not apply and we can observe them using the same given experimental arrangement with a single measurement (also see [1, 46, 62, 112, 117] in this regard), there by showing that unrefined complementarity principle within SQM is just a consequence of uncertainty principle. And they [12, 73] support NPI because tunneling cannot be satisfactorily explained by considering photon as a classical particle like in [46]. Note that even in Eq. (10), using a single measurement outcome (which of course contains many photons), we infer about manifestation of both wave and particle natures in the premeasurement scenario on time scales $\gg \bar{\tau}$. Hence the observables corresponding to them must be commuting (see supplementary material for details).

The experiment in [103] claims to have observed the presence of entanglement between c.m d.o.f of atom and the photon it emits spontaneously. But those observations can be explained within NPI, even when there is no entanglement between c.m d.o.f of atom and the photon it emits, via momentum and energy conservation alone. And in [36], entanglement between c.m d.o.f of atom and the photon it scatters is assumed a priori. Moreover, even within NPI, there will be entanglement between atom (c.m d.o.f) and the photon it emits, provided $\bar{\mathcal{T}}_{spo} \gg \bar{\tau}$. Hence these experiments do not throw any light on NPI.

In the experiments [40, 111], photo-detector was kept very close to the atoms. In this case, excluding the possibility of very first scattered photon (in the detection region) being unentangled with the atom, there is no chance of getting photons from both locations 0 and 1. Moreover, the experiments [40, 111] had not scattered photons off of a single atom in spatial superposition (which is necessary for the direct test of NPI) but had used an ensemble of atoms. Further, to test NPI, the condition $\bar{\mathcal{T}}_{spo} \lesssim \bar{\tau}$ should be satisfied.

Using a localized wave packet (i.e., a square integrable state vector) describing the physical state of an hopping particle, we can satisfactorily explain the photoelectric effect [15], Compton scattering [15], and violation of the symmetries corresponding to charge conjugation and parity in weak interaction [87] within NPI as well, in spite of dynamic delocalization of the particle (see supplementary material for details).

No connection between discontinuous jumps of NPI and GRW: In Ghirardi-Rimini-Weber model (GRW) of discontinuous spontaneous localization [72], the state-vector/wave-function evolves discontinuously in time (i.e., jumps/hops instantaneously) as a consequence of wave function collapse [71, 123] induced by universal classical-force noise [153]. And in the limit of mean frequency of jumps$\to \infty$, GRW $\to$ continuous spontaneous localization model (CSL) [71]. Whereas in NPI, an hopping particle evolves discontinuously in time which is demanded by the unification of wave and particle, and also by the experimental facts like absence of electrostatic self repulsion of electron in an Hydrogen atom (which is implied by the linearity of SE) etc. Hoppings take place by virtue of quantum nonlocality which involves no classical force. And hence hoppings (and hence N) in itself cannot induce wave function collapse. And in the limit $\bar{\nu} \to \infty$, NPI $\to$ UQW. This shows that NPI and GRW (and hence CSL) are independent of each other. This is further justified by the fact that in Eq. (11), there is no additional noise term (Weiner process) along with the Hamiltonian like in CSL [71]. (See supplementary material for details.) Consequently the experiments which disfavor CSL [30, 31, 61, 94, 153] and gravity induced collapse model [48, 61, 94], favor the fact that within NPI there seems to be no need of objective [72]/subjective [87, 105, 155, 160] wave function collapse to account for the macroscopic world we observe, as explained earlier. Hence there can be no connection between the discontinuous jumps of NPI and that of GRW.

Discussion and conclusion: Principle of unification of wave and particle allows only a particle without (but not with) hidden variable i.e., a nonlocal-hopping particle (because existence of a particle with hidden variable guided by a quantum wave implies existence of two worlds viz., classical and quantum, with contradictory properties). There are already strong experimental evidences (like the absence of electrostatic self-repulsion of the electron in an Hydrogen atom (which is implied by the linearity of Schrödinger equation) combined with Schrödinger's charge density hypothesis [67], very short range nature of strong nuclear force combined with gravitationally induced neutron interference [43] etc.) in favor of the same i.e., nonlocal-hopping particle interpretation (NPI). Hence, when we put a single internally highly-excited massive atom/molecule in spatial superposition of two distinct locations 0 and 1, there are strong and compelling reasons to look for photons coming from both locations. By this we can measure a single unknown quantum state (this in turn implies signaling i.e., superluminal communication). This is further supported by Bell and Aharonov-Bohm nonlocalities, and the signaling predicted within the frequentist-inspired quantum mechanics [101, 102]. Further, existence of a nonlocal-hopping particle may point towards the possible existence of an underlying unmanifest and absolutely empty quantum nonlocal wave/field from which spacetime itself (and also matter) might be emerging (otherwise we cannot make sense of nonlocal-hopping). Hence unification of wave/field and particle appears to be at a more fundamental level than unification of different forces. Further, within NPI, we may account for the macroscopic world we observe, without the necessity of wave function collapse or many worlds interpretation. Further note that when we consider evolution in spacetime (local phenomena) then speed of light is the limit. But nonlocal phenomena, by very definition, is not an evolution in spacetime, and hence the speed of light limit do not apply. Further note that signaling is predicted even by a qubit traversing a CTC [24], and also by PT-symmetric non-Hermitian quantum mechanics [106, 162]. Further, the concept of a particle without hidden variable (which is consistent with the known experimental results) and hence the scheme of unification of wave and particle proposed here, appears to be unique.

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